Iterative-Transform Phase Diversity: An Object and Wavefront Recovery Algorithm

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Abstract: Presented is a solution for recovering the wavefront and an extended object. It builds upon the VSM architecture and deconvolution algorithms. Simulations are shown for recovering the wavefront and extended object from noisy data.

OCIS codes: (100.5070) Phase retrieval; (100.1830) Deconvolution; (010.7350) Wave-front sensing;

1. Introduction

Phase retrieval is an algorithm used to recover the exit-pupil wavefront and thus, to measure or recover the deviation of the system from the desired optical system, [1]. The Variable Sampling Mapping (VSM) is a modification to the the Hybrid Diversity Algorithm (HDA) [2], which is an iterative-transform phase-retrieval algorithm based on the Misell-Gerchberg-Saxton algorithm [3,4]. HDA is unique because it has an adaptive diversity kernel, which increases the dynamic range of earlier work and solves an image registration problem. VSM is a method for enforcing a more general amplitude constraint, which allows one to incorporate in the forward and backward model various source illuminations, detector properties, and noise environments, e.g. broadband, under-sampled, extended objects, jitter, etc. [5]. VSM, like other phase-retrieval algorithms, makes use of images recorded when the optical system is illuminated by a point source or a known object. Accurate estimation of the wavefront requires good knowledge of the irradiance distribution of the source, and a model for opto-mechanical disturbances (like jitter) that were present during the recording of the images. This assumption makes the separation of various image-plane convolution kernels problematic, for example jitter, extended objects, etc. In most situations, the jitter or the object being imaged is unknown or estimated, and this limited knowledge impacts the accuracy of estimating the exit-pupil wavefront. Thus, one must be able to recover the point-source convolution kernel in addition to the wavefront. Phase diversity and blind-deconvolution are well-established object and wavefront estimation techniques, [6-9]. To date, most solutions to the phase-diversity problem are parametric, e.g. Zernikes, point-by-point, etc. Ultimately minimizing an error-metric function using non-linear optimization to find a solution. These solutions can be very computationally demanding, requiring an error metric to be evaluated hundreds or thousands of times.

2. Algorithm Overview

The purpose of the algorithm described here is to build upon VSM and various deconvolution algorithms, to estimate the unknown object shape and pupil wavefront. We show in this paper that VSM, when provided the convolution kernel (object, jitter, etc) and additional optical parameters, it can accurately recover the exit-pupil wavefront. There are numerous approaches to deconvolution, suitable for a wide variety of applications, [10-12]. By combining VSM and deconvolution, one is able to simultaneously recover the exit pupil wavefront and the image-plane convolution kernel. Figure 1 shows a block diagram of the algorithm. The algorithm starts with an initial estimate for the object and

Figure 1: Iterative-Transform Phase Diversity

phase. Then, VSM phase retrieval estimates an improved wavefront, and a model of the of the point-spread function (PSF). The PSF is deconvolved with the data to produce an improved estimate of the object. The process is repeated until convergence criteria are met.
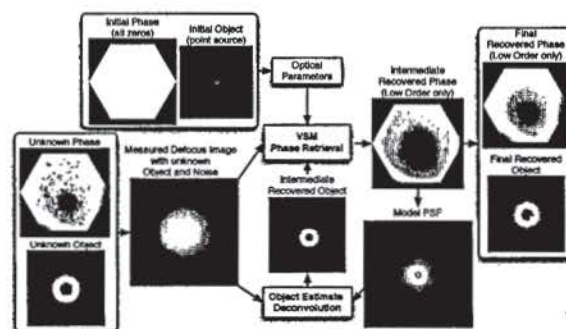
3. Simulations

The Hybrid Diversity Algorithm (HDA) [2] for phase retrieval is the iterative-transform "engine" used with VSM for this study. Shown in Figures 2 and 3 are the results for an example simulation for recovering the wavefront and the extended object. For this simulation, the extended object is a two-dimensional Gaussian function. Furthermore, four defocused images were used as input to the phase-retrieval algorithm, with various noise sources and uncertainties modeled in the simulated data. For this single realization, the major-axis length-scale σ_A and the minor-axis length σ_B are recovered with an error of 8% and 3%, respectively, and the low-order Zernikes (defined as

Zernikes 4-21) of the wavefront are recovered with a total error of $\lambda/118$ RMS, as shown in Figure 3. The full wavefront recovery error is dominated by high spatial frequencies in the wavefront; these high-frequency errors are caused by the noise sources in the detector, particularly visible in the wings of the MTF plots shown in Figure 2. For this simulation, two additional cases where considered, but are not shown graphically: 1) The extended object was modeled incorrectly as a point source and not updated during retrieval, and only the wavefront was retrieved, and 2) perfect knowledge of the extended object was provided to phase retrieval, and only the wavefront was retrieved. These two cases give a context for the results described above and demonstrate the importance of object recovery within phase retrieval. For case 1, the low order wavefront was recovered with an error of $\lambda/38$ RMS and this error was dominated by power, astigmatism, and spherical. For case 2, the low order wavefront error was recovered with an error of $\lambda/235$ RMS.
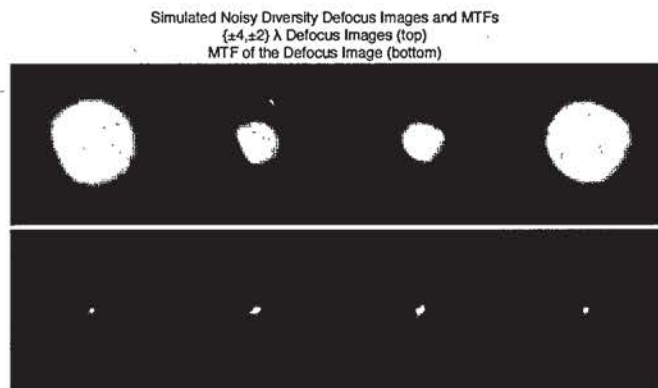

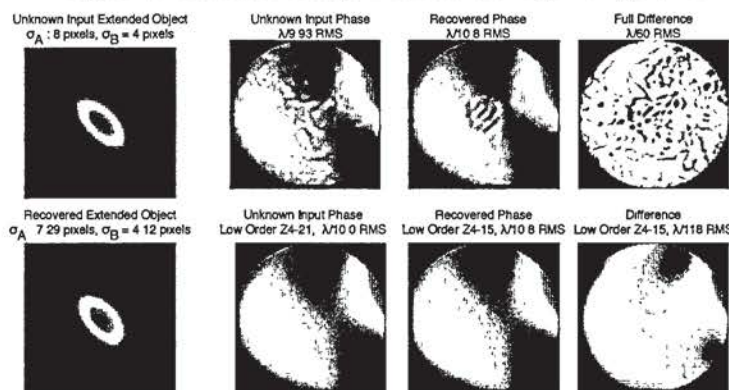Figure 2: Simulation Image Data and Corresponding MTFs

Figure 3: Unknown and Recovered Extended Object and Phase

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